定理 0.1 (6.4) *任给真值赋值σ，任给公式β 和γ，都有:*

(1) $\beta^\sigma \vdash (\neg\beta)^\sigma$

(2) $\{\beta^\sigma, \gamma^\sigma\} \vdash (\beta \to \gamma)^\sigma$

Proof. (2)如果$\sigma(\beta \to \gamma)$=0,则$\sigma(\beta)$=1且$\sigma(\gamma)$=0, 所以$(\beta \to \gamma)^\sigma = \neg(\beta \to \gamma)$, $\beta^\sigma = \beta$, $\gamma^\sigma = \neg\gamma$. 易证$\{\beta, \neg\gamma\ \} \vdash \neg(\beta \to \gamma)$. 即，$\{\beta^\sigma, \gamma^\sigma\} \vdash (\beta \to \gamma)^\sigma$.

如果$\sigma(\beta \to \gamma)$=1,则$(\beta \to \gamma)^\sigma = \beta \to \gamma$. 又分两种情况. 当$\sigma(\beta)$=0 时，有$\beta^\sigma = \neg\beta$, 从$\neg\beta \vdash \beta \to \gamma$ 得$\{\beta^\sigma, \gamma^\sigma\} \vdash (\beta \to \gamma)^\sigma$. 当$\sigma(\gamma) = 1$ 时，有$\gamma^\sigma = \gamma$, 从$\gamma \vdash \beta \to \gamma$ 也得$\{\beta^\sigma, \gamma^\sigma\} \vdash (\beta \to \gamma)^\sigma$. ■

定理 0.2 (6.5) *任给$\alpha \in \Phi_n$(即，α中的命题变元都在$p_1, p_2, \ldots, p_n$), 任给真值指派σ，都有$\{p_1^\sigma, p_2^\sigma, \ldots, p_n^\sigma\} \vdash \alpha^\sigma$.*

Proof. 施归纳于Φ_n的结构.
(1) $\alpha = p_i (i = 1, 2, \ldots, n)$. $\alpha^\sigma = p_i^\sigma$, 当然有, $\{p_1^\sigma, p_2^\sigma, \ldots, p_n^\sigma\} \vdash \alpha^\sigma$.
(2) $\alpha = \neg\beta$. 由归纳假设$\{p_1^\sigma, p_2^\sigma, \ldots, p_n^\sigma\} \vdash \beta^\sigma$. 再由定理6.4得, $\beta^\sigma \vdash (\neg\beta)^\sigma$,最后得$\{p_1^\sigma, p_2^\sigma, \ldots, p_n^\sigma\} \vdash \alpha^\sigma$.
(3) $\alpha = \beta \to \gamma$. 由归纳假设$\{p_1^\sigma, p_2^\sigma, \ldots, p_n^\sigma\} \vdash \beta^\sigma$ 和$\{p_1^\sigma, p_2^\sigma, \ldots, p_n^\sigma\} \vdash \gamma^\sigma$. 再由定理6.4得, $\{\beta^\sigma, \gamma^\sigma\} \vdash (\beta \to \gamma)^\sigma$, 最后得$\{p_1^\sigma, p_2^\sigma, \ldots, p_n^\sigma\} \vdash \alpha^\sigma$.

■

定理 0.3 (6.5) *所有重言式都是内定理.*

Proof. 任给重言式，可以找到n, 使得$\alpha \in \Phi_n$. 又因为α是重言式，所以任给真值指派$\alpha^\sigma = \alpha$.
归纳证明任给真值指派σ, 任给$1 \le k \le n$, 都有$\{p_1^\sigma, \ldots, p_n^\sigma\} \vdash \alpha$.
(1) k=1. 就是定理3.5.
(2) k=m+1时. 任给真值指派σ, 取$\sigma_1 = \sigma(m, 1)$, $\sigma_2 = \sigma(m, 0)$. 由归纳假设，对σ_1和σ_2有$\{p_m^{\sigma_1}, \ldots, p_n^{\sigma_1}\} \vdash \alpha$ 和$\{p_m^{\sigma_2}, \ldots, p_n^{\sigma_2}\} \vdash \alpha$. 这就是$\{p_m^\sigma, p_{m+1}^\sigma, \ldots, p_n^\sigma\} \vdash \alpha$ 且$\{\neg p_m^\sigma, p_{m+1}^\sigma, \ldots, p_n^\sigma\} \vdash \alpha$. 由反证法第三形式，$\{p_{m+1}^\sigma, \ldots, p_n^\sigma\} \vdash \alpha$.
(3) 当k=n时，就是任给真值指派σ,有$p_n^\sigma \vdash \alpha$. 取$\sigma_1(p_n) = 1$, 取$\sigma_2(p_n) = 0$，则$p_n \vdash \alpha$ 且$\neg p_n \vdash \alpha$. 由反证法第三形式得$\vdash \alpha$.

■